

## THE RELATION BETWEEN INTERFACIAL AREA AND THE RATE OF ENERGY DISSIPATION IN BUBBLE COLUMN

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The interfacial area in bubble columns can be successfully correlated by means of the relations derived on the assumption of isotropic turbulence in the liquid phase. It turns out that for the formation of the interfacial area the controlling effect is that of the total rate of energy dissipation in both phases.

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The overall rate of a reaction between a dissolving gas and a liquid reactant in the two-phase system and the interfacial area in particular usually markedly depend on the hydrodynamic parameters of the gas-liquid mixture. An exception to this are reactions with extremely low reaction rate constants taking place in the bulk liquid phase. However, the so far gathered information on the hydrodynamics of heterogeneous systems indicate that the determination of interfacial area as well as its correlation is by no means a simple matter. The extent of the interfacial area depends on a number of parameters affecting the structure of the two-phase bed, namely both on the physico-chemical parameters of the system and the operating conditions. Correlation of all these parameters with possible effect on the interfacial area though is difficult. Irreproducible quantities of surfactants often present only in traces, ionic strength, density of the surface charge, volume and valency of ions of dissolved solutes, diameter and depth of the bed of liquid are typical examples of the parameters that may markedly affect the structure of the heterogeneous bed and the interfacial area. It is thus not surprising that the data on the extent of interfacial area, in our case for bubble columns, published by different authors significantly differ. A critical analysis usually reveals that the published value of the interfacial area for given conditions and reactor holds only for these very conditions and that an extrapolation to other geometrical arrangements is unreliable. Yet, the design and calculation of chemical reactors for two-phase systems calls for at least an approximate knowledge of the interfacial area and chiefly about the changes with the operating conditions controlled mostly by the flow rates of phases. This need strongly stimulates the search for at least approximate approaches until more exact methods have become available. Instead of attempts for "general" correlations, which in view of the above seem to have very little chance for success, we have resorted to the derivation of the relations enabling the effect of some hydrodynamic parameters to be assessed. These relations are based on model concepts of mass transfer mechanism and their ultimate goal is to permit data transfer from bench to large-scale equipment.

In this paper we have attempted to critically evaluate the contemporary concepts starting from the theory of isotropic turbulence which lead to the correlations for the interfacial area and to ascertain the dependence of the interfacial area on principal

hydrodynamic parameters of the bubble column. In order to test these relations we have used both our own experimental data as well as those published in the literature where the authors measured also the porosity of the bed apart from the interfacial area.

### THEORETICAL

Kolmogorov's theory of isotropic turbulence was applied successfully first by Kolář<sup>1,2</sup>, and later by Calderbank<sup>3</sup> in order to derive a relation for the coefficient of mass transfer. This concept can, in principle, be used also to derive a correlation for the interfacial area in a heterogeneous bed of two fluids. Intuitive assumption of the existence of direct proportionality between the total rate of energy dissipation,  $e$ , related to a unit volume of the gas-liquid mixture on one hand and the extent of interfacial area on the other hand has led to the following relation<sup>4</sup>

$$a \sim v_g(1 - \varepsilon). \quad (1)$$

This proportionality fits the course of the interfacial area with variable flow rate of gas through a bubble column of solution of an electrolyte. Its derivation, however, is not quite consistent. A comparison of various types of contact apparatuses from the viewpoint of the dependence of the interfacial area on the overall rate of energy dissipation within the heterogeneous mixture has been attempted by Nagel and co-workers<sup>5</sup>. These authors have thus presented the so far most advanced application of the theory of isotropic turbulence to the problem of correlating interfacial area in gas-liquid systems. This approach forms the starting point also in this work with the alternative replacement of the rate of energy dissipation by that dissipated only in the liquid.

In the derivation it is assumed that a part of energy dissipated in the heterogeneous bed is dissipated in order to create interfacial area and that between both quantities there is a strong correlation. The rate of energy dissipation in a bubble bed can be easily determined provided the porosity of the bed is known for a given velocity of gas. The derivation further assumes the existence of isotropic turbulence in the bulk liquid which is the continuous phase in bubble columns<sup>3</sup>. A qualitative description of the conditions prevailing near the interface has been described sufficiently in the original papers. Eventual effect of interfacial turbulence will be regarded as insignificant.

On adopting Nagel's<sup>5</sup> approach leading to the expression for the expected mean size of a gas bubble in the bed,  $\bar{d}_B$ , and on expressing the turbulent shear stress in terms of the rate of energy dissipation in accord with the theory of isotropic turbulence<sup>1-3,5-7</sup>, we obtain

$$\bar{d}_B \sim (\sigma/\rho_L)^{0.6} e_d^{-0.4}. \quad (2)$$

$e_d$  is the rate of energy dissipation related to a unit mass of liquid,  $\sigma$  is the interfacial tension and  $\rho_L$  is the density of liquid. Assuming direct proportionality between the equivalent diameter of a spherical bubble,  $d_c$ , and the mean bubble diameter,  $\bar{d}_B$ , we may write for spherical isolated bubbles

$$a = (6/d_c) \varepsilon. \quad (3)$$

For the specific interfacial area we thus get

$$a \sim (\rho_L/\sigma)^{0.6} e_d^{0.4} \varepsilon, \quad (4)$$

where  $\varepsilon$  is the mean porosity of the bed. Up to this point the derived dependence between the overall rate of energy dissipation and the specific interfacial area is practically identical with the relations published by Nagel<sup>5</sup> and Calderbank<sup>3</sup>.

For the overall rate of energy dissipation in bubble columns exhibiting large hold-up of liquid related to a unit mass of liquid one can derive the following relation

$$e_d = v_g g. \quad (5)$$

For the specific interfacial area we can then write

$$a \sim v_g^{0.4} \varepsilon, \quad (6)$$

where  $v_g$  is the gas velocity in the column and  $a$  is the specific interfacial area related to a unit volume of the bed.

From the viewpoint of the assumptions made in the derivation of the turbulent shear stress there is a possibility of substituting in Eq. (4) instead of the overall rate of dissipation,  $e_d$ , only the energy dissipated in the liquid,  $e_{WD}$ , which forms only a part of the rate  $e_d$ .

For the rate of energy dissipation only in the liquid phase (related to a unit mass of liquid) Kolář<sup>2</sup> has derived a relation which can be written, after some arrangement, as

$$e_{WD} = v_g \frac{\mu_g}{\mu_L} \frac{1 - \varepsilon}{\varepsilon}, \quad (7)$$

where  $\mu_g$  and  $\mu_L$  are the viscosities of gas and liquid respectively. Substituting Eq. (7) into (4) we obtain

$$a \sim (\rho_L/\sigma)^{0.6} (\mu_g/\mu_L)^{0.4} v_g^{0.4} (1 - \varepsilon)^{0.4} \varepsilon^{0.6}. \quad (8)$$

For the studied gas-liquid system and the given geometry of the bed the effect of the

flow rates of phases on the hydrodynamic character of the bed is expressed, according to Eq. (8), explicitly through the gas velocity and implicitly through the porosity of the bed. Porosity of bubble columns is virtually independent of the flow rate of liquid but depends on the flow rate of gas. The simplest expression has been presented by Reith<sup>8</sup>

$$\varepsilon = v_g / (2v_g + c). \quad (9)$$

In contrast to the original opinion the constant  $c$  is not universal and for various types of solutions ranges over a rather wide interval. Nevertheless, Eq. (9) approximates the true course of porosity with reasonable accuracy.

Substituting Eq. (9) into (8) we obtain

$$a \sim (v_g + c)^{0.4} \varepsilon \quad (10a)$$

or

$$a \sim (v_g + c)^{0.4} v_g / (2v_g + c). \quad (10b)$$

For small gas velocities when we may write that  $v_g < c$  and thus  $v_g$  in Eq. (10a) may be neglected (for  $v_g \leq 3 \text{ cm s}^{-1}$  with an error of 10%) and the course of the values of the interfacial area is proportional to the porosity

$$a \sim \varepsilon. \quad (10c)$$

In Eqs (10) we have omitted the terms characterizing physico-chemical macroparameters of the system because the set is incomplete and includes so far unknown parameters.

A comparison of Eqs (3) and (10a) offers the following physical interpretation: In region of low gas velocities the diameter of bubbles remains constant with increasing flow rate of gas and the interfacial area and porosity grow as a consequence of increased absolute amount of bubbles of identical size present (or as a consequence of increasing frequency of bubble formation in gas distributor) which is in agreement with earlier findings<sup>9-12</sup> and confirms our considerations. Eqs (8) and (10) were used to correlate experimental data for two phase systems.

## DISCUSSION

Fig. 1 shows the published\* as well as our own data on specific interfacial area according to the correlation (8) ref.<sup>8,13-16</sup>. Except for the inherent scatter of data given

\* The selected published data were read off the appropriate graphs in the original papers or computed from recommended correlations.

by the method of measuring the interfacial area the agreement is remarkable. The data essentially appear on straight lines passing through origin. Fig. 2 shows our data according to correlation (10) which enable the dissipated energy to be expressed by Eq. (7). The results are even more favourable than those corresponding to the correlation (6) with the traditional expression of the rate of energy dissipation as they were published earlier<sup>13</sup>. The answer which way of correlation is preferable is not unambiguous for the case of specific interfacial area for the rate of energy dissipation expressed in Eq. (7) remains almost unaffected by a changing flow rate of liquid (in contrast to the rate of energy dissipation given by Eq. (5)). This may suggest that the controlling effect for the formation of the interfacial area is that of energy dissipated in both phases. However, the energy dissipated in liquid remains no doubt adequate for processes taking place in the liquid phase, *i.e.* mainly for the mechanism of mass transfer proper. It is noted that by respecting the physico-chemical macro-parameters in Eqs (6) and (8) the published data cannot be concentrated on a single straight line even for the same column diameter because the physico-chemical macro-parameters of various systems of electrolytes are virtually identical.

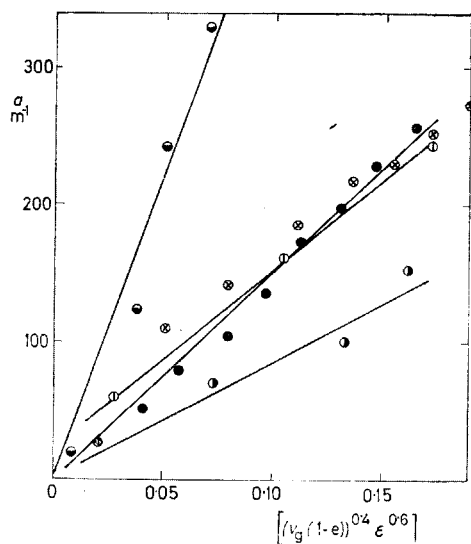


FIG. 1

Specific Interfacial Area as a Function of Gas Velocity and Porosity according to Eq. (8)

● Reference<sup>13</sup>, ●<sup>14</sup>, ⊗<sup>8</sup>, ⊙<sup>15</sup>, ⊖<sup>16</sup>

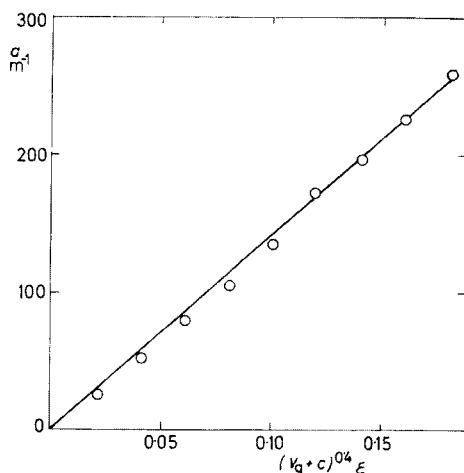


FIG. 2

Specific Interfacial Area as a Function of Gas Velocity and Porosity according to Eq. (10a)

Absorption of pure CO<sub>2</sub> into water solution of ammonia in a 150 mm in diameter column (ref.<sup>13</sup>).

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